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Report No. 292

CALIBRATION OF DOPPLER RADAR USING TUNING FORKS

by

Lester Martin Lendrum

September 1, 1968

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DEPARTMENT OF COMPUTER SCIENCE
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN · URBANA, ILLINOIS

Report No. 292

CALIBRATION OF DOPPLER RADAR USING TUNING FORKS*

by

Lester Martin Lendrum

September 1, 1968

Department of Computer Science
University of Illinois
Urbana, Illinois 61801

* Submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering in the Graduate College of the University of Illinois, 1968.

ACKNOWLEDGMENT

The author wishes to thank Professor T. A. Murrell for his patience and guidance in the preparation of this paper. The generosity of Muni Quip Corporation in supplying the Radar Timer was greatly appreciated. The author also wishes to express appreciation to the University Research Board for providing funds for computer computations. Special thanks go to Miss Marian Buch whose help in typing and proof-reading the preliminary drafts was invaluable.

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1. INTRODUCTION

There are in use today many radar units which depend on the Doppler shift in frequency to measure the velocity of a moving object. In most instances, such as measurement of speeds of vehicles on a highway, the difference frequency falls in the audio range (Appendix A). When this is the case, a convenient and commonly used method of calibration consists of placing a vibrating tuning fork close to the antenna and in the direct path of the radiation, and adjusting the conversion circuits to bring the indicated speed into agreement with the reading corresponding to the tuning fork frequency. In spite of the advantages afforded by this method, no investigations in this area are available.

The purpose of this study is to evaluate the reliability of tuning fork calibration. In this respect, the operation of the radar unit will be analyzed for normal use in regard to reading speeds and for calibrating conditions. Therefore, the objectives of this study are to:

1. Analyze the process of speed measurement by Doppler radar.
2. Analyze the mixing and detecting processes in the particular radar unit under study.
3. Predict the theoretical detector output wave forms for the following cases:
   a) where the reflecting object is moving at a constant rate of speed.
   b) where the reflecting object is a vibrating tuning fork.
4. Correlate the theoretical detector outputs with actual detector outputs for the case of a vibrating tuning fork.

5. Show that the signal generated by a vibrating tuning fork is a satisfactory calibration signal for Doppler radar units.

2. TARGET SPEED MEASUREMENT BY DOPPLER SHIFT

In normal usage, the radar beam, usually shaped to a width of a few degrees by the antenna assembly, is aimed at the target and radiation from the target is received by the same or another antenna.

As shown in the block diagram of Figure 1, a small portion of the transmitted frequency is fed to a mixer, where it is added to the signal received from the moving object. For electromagnetic radar units, operating at higher 10,525 mHz. or 2,450 mHz., the radio frequency energy is fed directly to the mixer, which consists of a length of waveguide or transmission line. The detector is usually a semiconductor crystal diode designed to respond to these high frequencies. To a good approximation, the output of the detector is proportional to the square of the envelope of the mixed wave (square-law detector).

The various waveforms of interest are shown in Figure 2. Mixing the two sinusoidal R.F. waves produces an interference pattern with envelope variations at the difference frequency, Δf. Although the envelope variation is not sinusoidal, (it is the square root of a sinusoidal), the square law detector will generate a sinusoidal output. The output of the detector is amplified and then converted into a square wave which is at the same frequency. The square wave is then clipped and differentiated to form a pulse train of period $\tau = 1/\Delta f$. The pulse train is then fed into the conversion circuit which produces a direct current proportional to the number of pulses per second and hence to the difference frequency, Δf. The square-wave convertor, the differentiator, and the pulse to direct current convertor are used to be certain that

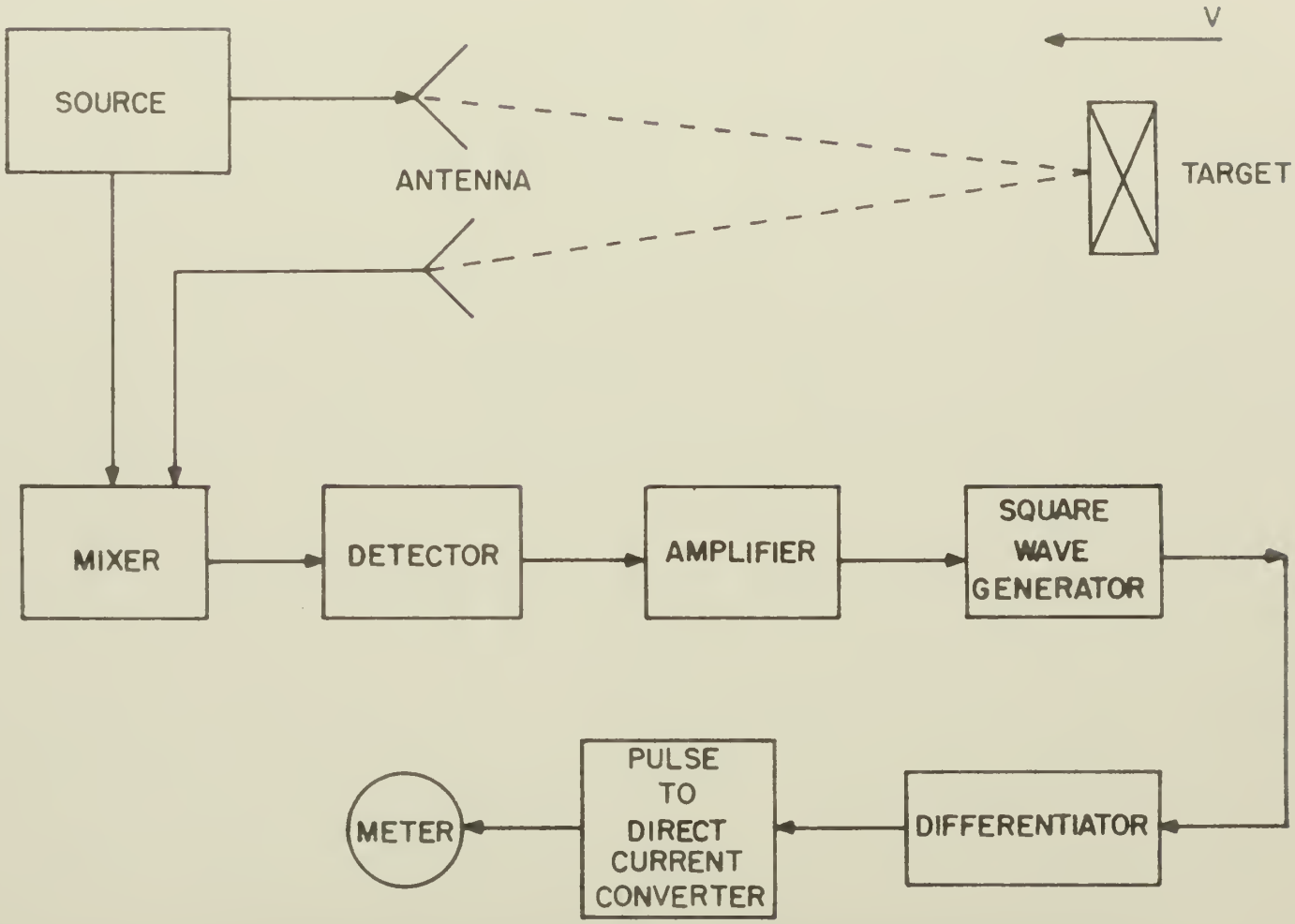

Figure 1. Block Diagram of Doppler Radar Unit.

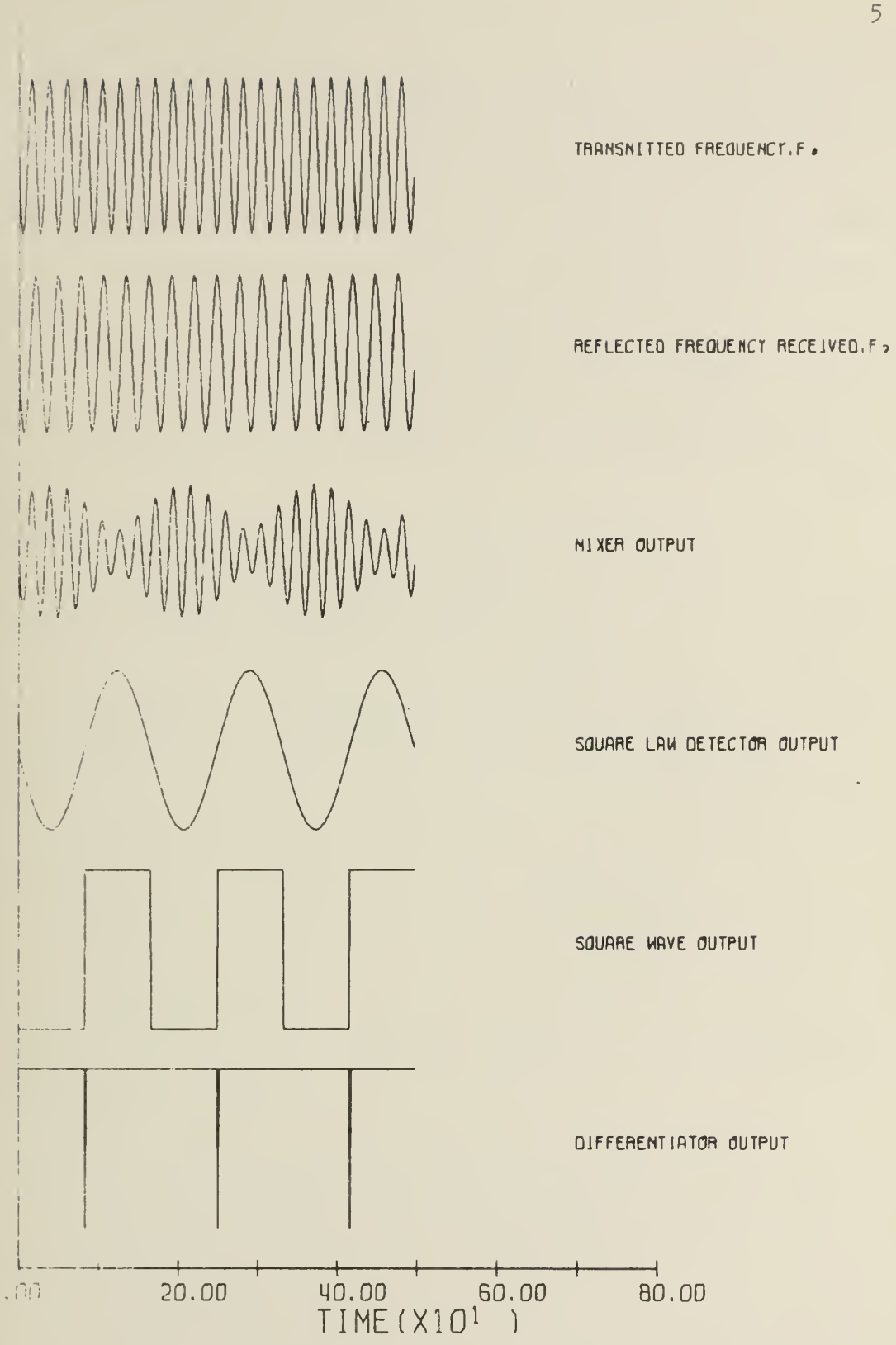

Figure 2. Waveforms of Doppler Radar Unit.

the speed indication is a function only of the difference frequency and not of the amplitude of the received signal.

Thus, using this method, the Doppler difference frequency Δf controls the current through a D.C. meter calibrated in miles per hour. By adjusting the circuit values, the meter can be calibrated to read 10 mph for a difference frequency of 314 Hz., 20 mph for a difference frequency of 628 Hz., etc. . Thus, the meter indicates the velocity of the moving object by measuring the frequency of the Doppler shift it causes.

Since it is the signal output from the amplifier section which contains the information of interest, the remaining circuitry will not be discussed. The relationship under consideration is that existing between the movement of the target and the output of the amplifier section. With this in mind, the mixing and detecting processes will be analyzed in detail.

3. ANALYSIS OF MIXING AND DETECTING PROCESSES

The particular radar unit under study operates at 10,525 mHz. The mixer in this case is a length of RG 52/W waveguide. The detector is a 1N23C semiconductor crystal diode.

3.1 Mixing

The dominant mode in rectangular guides such as RG 52/W is $TE_{1,0}$.[1] The transverse electric field is the quantity of interest since diode detectors respond to this field only. The electric field of a $TE_{1,0}$ mode in a waveguide is given by:[2]

$$E = K\,\omega\, e^{j(\omega t + \theta)} e^{-j\sqrt{(\pi/a)^2 - \omega^2\mu\epsilon}\, z} \qquad (1)$$

where K is a constant depending on the strength of the signal, θ is the time phase angle with respect to some reference, ω is the radian frequency of the signal, μ is the permeability of air, ϵ is the permittivity of air, a is the width of the waveguide, and z is the distance along the axis of the guide. This wave is propagating in the positive z-direction.

The mixer consists of an antenna coupled to a shorted section of waveguide as shown in Figure 3.

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1. E. C. Jordan, Electromagnetic Waves and Radiating Systems, p. 289.
2. Ibid., p. 267.

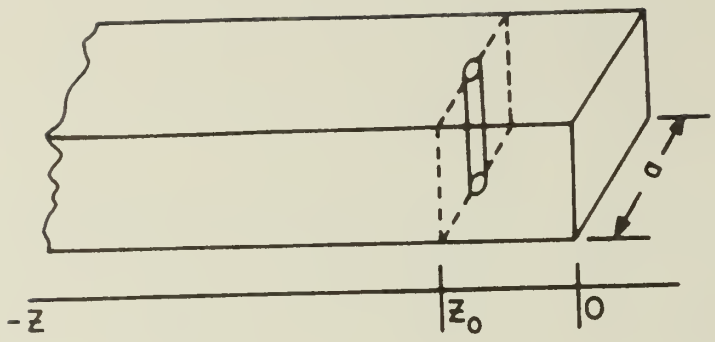


Figure 3. Mixer and Detector

If an R.F. signal is received by the antenna, a $TE_{1,0}$ wave propagating to the right is set up in the guide. Let the electric field of this wave be E_I. Since the waveguide is shorted at z = 0, the incident wave E_I is reflected totally; the reflected wave is E_R. The incident electric field may be represented by

$$E_I(t,z) = K_0\ \omega_0\ e^{j(\omega_0 t + \theta_0)}\ e^{-j\sqrt{(\pi/a)^2 - \omega_0^2 \mu\epsilon}\ z} \tag{2}$$

where K_0 depends on the strength of the signal, and θ_0 is the time phase angle referenced to the R.F. source.

The reflected wave, E_R, is propagating in the negative z-direction and its magnitude is the negative of that of E_I, since $E_I(t,0) + E_R(t,0) = 0$ (a conducting surface). Thus

$$E_R(t,z) = -K_0\ \omega_0\ e^{j(\omega_0 t + \theta_0)}\ e^{j\sqrt{(\pi/a)^2 - \omega_0^2\ \mu\epsilon}\ z} \tag{3}$$

The incident and reflected electric field form a standing wave pattern in the guide. The resultant electric field in the guide is the sum of the incident and reflected fields.

$$E_{Resultant}(t,z) = E_I(t,z) + E_R(t,z)$$

$$= -2jK_0\omega_0 e^{j(\omega_0 t+\theta_0)} \sin\sqrt{(\pi/a)^2 - \omega_0^2\mu\epsilon}\, z \qquad (4)$$

The diode detector is mounted in the guide at a fixed positon z_0, as shown in Figure 3. This position is chosen in the design of the unit such that sin is fairly close to unity. Since the electric field of interest is that at the position of the detector, the above equation may be simplified to:

$$E_{Resultant}(t) = K' e^{j(\omega_0 t+\theta_0)} \qquad (5)$$

This is the electric field at the detector for a single incident wave. If more than one incident wave is present, superposition is used to find the total electric field in the guide.

In the following calculations, all frequency shifts will be treated as continuous phase shifts; that is, a signal having $e^{j\omega_1 t}$ time dependence will be written as having $e^{j(\omega_0 t + \theta(t))}$ time dependence, where $\theta(t) = (\omega_1-\omega_0)t$ in this case (Appendix B).

In Doppler type radar units using a waveguide mixer-detector as described above, energy from the source is fed to the mixer-detector by two paths (Figure 4). A small reflector, R, is build into the antenna assembly to send a small portion of the source energy into the mixer-detector (path A). This causes a standing wave to be set up in the

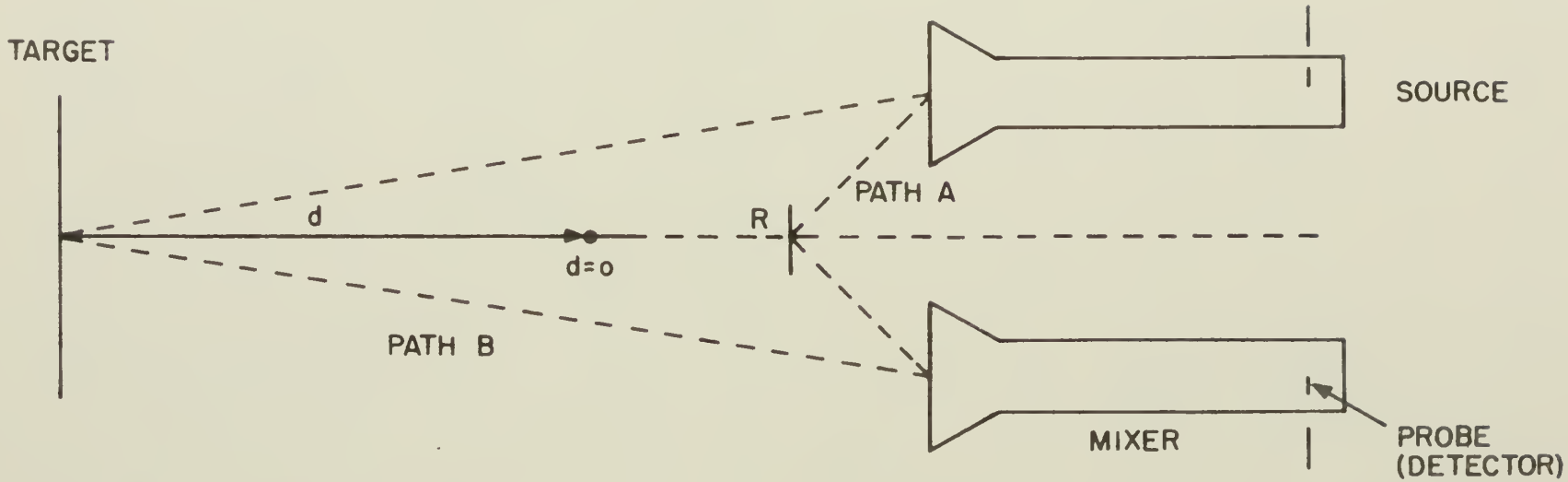


Figure 4. Antenna-Target Relationship.

mixing guide and the resulting electric field E_1 is of the same frequency ω_0, as the source.

$$E_1(t,z_0) = K_1 e^{j(\omega_0 t+\theta_0)} \tag{6}$$

where θ_0 is the fixed phase angle of E_1 with respect to the source which is due to the propagation delay over the fixed path A.

When a target is present, a portion of the source energy is reflected into the mixer-detector along path B. This causes a second standing wave in the guide,

$$E_2(t_1 z_0) = K_2 e^{j(\omega_0 t+\theta)} \tag{7}$$

where θ varies if the length of path B varies. In fact, $\theta = \theta_c + \frac{4\pi d}{\lambda}$ where θ_c is the phase angle between E_2 and the source when $d = 0$, λ is the wavelength corresponding to angular frequency ω_0, and d is the distance measured along the axis of the antenna system. Thus, the total electric field in the guide is $E_1 + E_2$ by superposition.

$$E_{total} = K_1 e^{j(\omega_0 t+\theta_0)} + K_2 e^{j(\omega_0 t+\theta_c + \frac{4\pi d}{\lambda})} \tag{8}$$

3.2 Detecting

The semiconductor crystal diode is positioned in the guide so that the electric field, E_{total} is along the axis of the diode holder. The electric field, E, and the diode current i, are related by the following formula: $i = A(e^{\alpha E} - 1)$, where A and α depend on the particular diode used. Diodes of this type are referred to as square-law since the filtered output signal is proportional to the square of the input signal.[3] Thus, in the case discussed here, the filtered output signal varies as the square of the amplitude of E_{total}.

A convenient method of representing the filtered detector output is vector representation.[4] Let the signal reaching the receiver directly from the source be taken as a reference. This signal will be represented by a vector e_1, of length $|E_1|$ and referenced horizontally to the right (Figure 5).

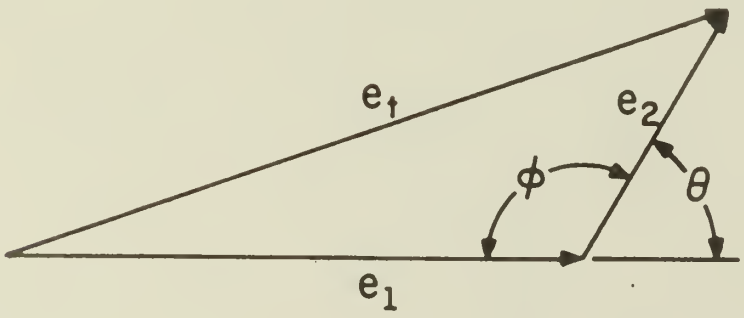


Figure 5.

3. R. V. Pound, Microwave Mixers, p. 19.

4. D. Luck, Frequency Modulated Radar, p. 12.

The signal returned from the target is represented by vector e_2, where $|E_2|$ is the magnitude of the returned signal and θ is the phase angle with respect to e_1. The vector e_t is the sum of e_1 and e_2. Let $e_1 = E_1(t,z_0)$, $e_2 = E_2(t,z_0)$ and $e_t = E_{total}$. Then $|E_1| = K_1$, $|E_2| = K_2$, and $\theta = \theta_e - \theta_0 + 4\pi d/\lambda$; see Equations 6, 7, and 8. Thus, the magnitude of vector e_t, $|E_t|$ corresponds to the magnitude of E_{total}. By the law of cosines,

$$|E_T|^2 = |E_1|^2 + |E_2| - 2|E_1|\ |E_2|\ \cos \Phi \tag{9}$$

Since

$\Phi = \pi - \theta$, $\cos \Phi = -\cos \theta$:

yielding

$$|E_T|^2 = |E_1|^2 + |E_2|^2 + 2|E_1|\ |E_2|\ \cos\ (\theta_c - \theta_0 + \frac{4\pi d}{\lambda}) \tag{10}$$

Since the detector output is the square of the envelope of the total electric field, the final result is

$$\text{DETECTOR OUTPUT} = |E_1|^2 + |E_2|^2 + 2|E_1|\ |E_2|\ \cos(\theta_c - \theta_0 + \frac{4\pi d}{\lambda}) \tag{11}$$

4. THEORETICAL DETECTOR OUTPUTS

Using the procedures developed in the last section, the detector outputs have been related to d, the distance between target and antenna. If this distance varies with time, then the detector output will also vary with time. In this section, the detector output is determined for two cases: first, the distance decreasing or increasing at a constant rate with respect to time (corresponding to the Doppler shift caused by a constant velocity target either closing or moving away); second, the distance varying sinusoidally about a fixed position (corresponding to reflection from a vibrating tuning fork).

4.1 Doppler Shift

Without loss of generality, only the case in which the target is moving directly away from the antenna assembly at a fixed velocity, v, is discussed. Thus, $d = d_0 + vt$ where d_0 is the distance between the antenna and target at $t = 0$. In Equation 11, the argument of the cosine becomes $\theta' + 4\pi vt/\lambda$ where $\theta' = \theta_c - \theta_0 + 4\pi d_0/\lambda$.

$$\text{DETECTOR OUTPUT} = C_1 + C_2 \cos(\theta' + 4\pi vt/\lambda) \tag{12}$$

This corresponds to the square law detector output as shown in Figure 2. The time dependence of the cosinusoid is $4\pi vt/\lambda$, which, since $\frac{2\pi}{\lambda} = f_0$, is equivalent to $\frac{2f_0 v}{c}$, the Doppler difference frequency if the direction of travel of the target is neglected (Appendix A). This result agrees with the discussion in Section II.

4.2 Vibrating Tuning Fork.

If the reflecting surface is a vibrating tuning fork, the distance, d, varies about a fixed point d_0. This oscillation is very nearly sinusoidal and therefore $d = d_0 + \delta \cos \omega_f t$, where δ is the maximum distance the tines of the tuning fork move from d_0 and ω_f is the angular frequency of the tuning fork oscillation. In Equation 11, the argument of the cosine becomes $\theta'' + 4\pi d_0/\lambda + (4\pi\delta/\lambda) \cos \omega_f t$, where $\theta'' = \theta_c - \theta_0$. Thus, the detector output is given by,

$$\text{DETECTOR OUTPUT} = C' + C'' \cos (\theta'' + 4\pi d_0/\lambda + (4\pi\delta/\lambda) \cos \omega_f t) \tag{13}$$

If $\frac{4\pi\delta}{\lambda}$ is small enough, a very close approximation of the expressions can be realized by expanding the cosine and sine functions in a power series.

$$\cos [(4\pi\delta/\lambda)\cos \omega_f t] = 1 - \frac{[(4\pi\delta/\lambda) \cos \omega_f t]^2}{2!} + \frac{[(4\pi\delta/\lambda) \cos \omega_f t]^4}{4!} - \ldots \tag{14}$$

$$\sin [(4\pi\delta/\lambda) \cos \omega_f t] = \frac{4\pi\delta}{\lambda} \cos \omega_f t - \frac{[(4\pi\delta/\lambda \cos \omega_f t]^3}{3!} + \frac{[(4\pi\delta/\lambda)\cos \omega_f t]}{5!} - \ldots \tag{15}$$

Using only the first two terms of Equation 14, and only the first term of Equation 15, the expression for the detector is approximated by,

$$\text{DETECTOR OUTPUT} \simeq C' + C''\left(1 - 4\pi^2\delta^2/\lambda^2\right)\cos\left(\theta'' + 4\pi d_0/\lambda\right) - C''\frac{4\pi\delta}{\lambda}\sin\left(\theta'' + 4\pi d_0/\lambda\right)\cos\omega_f t - C''\frac{4\pi^2\delta^2}{\lambda^2}\cos\left(\theta'' + 4\pi d_0/\lambda\right)\cos 2\omega_f t \qquad (16)$$

The true expression for the detector output involving Bessel functions is given here for completeness:

$$\text{TRUE DETECTOR OUTPUT} = C' + C''\cos\left(\theta'' + 4\pi d_0/\lambda\right)\left[J_0\left(4\pi\delta/\lambda\right) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(4\pi\delta/\lambda\right)\cos\left(2k\omega_f t\right)\right] - C''\sin\left(\theta'' + 4\pi d_0/\lambda\right)\left[2\sum_{k=0}^{\infty}(-1)^k J_{2k+1}\left(4\pi\delta/\lambda\right)\cos\left[(2k+1)\omega_f t\right]\right] \qquad (17)$$

Actually, the coefficients of the first and second harmonic terms are very close in the two expressions. The only difference between Equations 16 and 17 resides in the higher harmonics, and for all practical purposes, the harmonics higher than the second are so small as to be unobservable for any reasonable value of δ. Thus, the formula of Equation 17 is sufficient for the purpose of this discussion.

Comparing the detector output for a target moving at a constant velocity (Equation 13) with that for a vibrating tuning fork (Equation 16), it is evident that the only significant difference is the second harmonic component of Equation 16. Thus, if it can be shown that the second harmonic component caused by the vibrating tuning fork does not cause incorrect indications on the output meter, then the tuning fork method of calibration must be considered valid. The following section describes experiments performed to show this.

5. EXPERIMENTAL VERIFICATION

The radar unit used for experimental work was manufactured by Muni Quip Corporation of Decatur, Illinois, (Radar Timer Series 64). The circuit diagrams of this unit are shown in Figures 6 and 7.

Two separate experiments were performed: first, measurement of the actual detector output when a vibrating tuning fork is placed in front of the antenna to verify the results of Sections II and III, especially Equation 16; second, determination of the maximum amount of second harmonic distortion which can be tolerated in the detector output without affecting the accuracy of the conversion circuitry. A secondary observation of the maximum displacement of the tines of the vibrating tuning forks was also made; however, this observation was made only to determine an upper bound on δ, and therefore no attempt was made to measure it precisely.

5.1 Measurement of Detector Output.

Although measurements at single points were made with actual tuning forks, it was necessary to measure the first and second harmonics of the detector output for many values of d_0, the distance between the target and antenna. It is desirable that the value of δ be constant throughout each set of distances d_0. This was impossible to achieve with a tuning fork since the oscillations are damped by the air surrounding the fork and by internal friction. To provide a vibrating surface which oscillated at a constant frequency and amplitude, a small metal disk was attached to the cone of a permanent magnet speaker and the speaker was excited by an oscillator-amplifier system. An optical

bench and micrometer positioning drive were used to maintain alignment and to determine the distance d_0 precisely (Figure 8).

The signal observed was that present at point A, Figure 6. This point was chosen because the signal there is unaffected by any external adjustment of the RANGE or CALIBRATE controls. The previous stages, TR1 and TR2, are frequency compensated amplifiers.

The signal was monitored with frequency selective VTVM's (Hewlett-Packard Model 302A Wave Analyzers), one adjusted to measure the first harmonic, V_1, of the speaker frequency, the other adjusted to measure the second harmonic, V_2. The magnitude of the first and second harmonics were recorded for 66 values of d_0, each d_0 differing from the previous one by 1/4 millimeter. This gave a total change in d_0 of 1.625 centimeters. A typical set of measured values is given in Table 1. A series of data sets like that of Table 1, were averaged and plotted by computor (Appendix C) along with the same quantities calculated from Equation 16. This plot is shown in Figure 9.

Difficulties in obtaining data resulted from radio frequency noise caused by electrical equipment in the laboratory and from stray reflections from the optical bench and speaker frame. To reduce the effect of stray reflections, the optical bench and the speaker assembly were masked with a microwave absorbing material. A small channel was made through the speaker assembly mask directly in front of the vibrating disk. As a result of these precautions, the only moving reflector in the radiation beam was the vibrating disk.

It will be noted that no measured voltage was less than 2 mV RMS. This residual voltage might be caused by noise voltage present in the system due to crystal noise and stray pick-up. However, the most likely cause of the failure of the second harmonic voltage to reach zero is a second harmonic component generated by the square-law detector. An ideal square-law detector will generate an output proportional to the square of the input, but if the detector is not perfectly square-law (most crystal detectors are not), harmonics of the input signal may also be generated.[1] Only 2.5 percent second harmonic generation would be required to give the results found experimentally.

After finding similar results for other frequencies and for other values of δ, it was apparent that Equation 16 accurately describes the physical situation.

---

1. H. C. Torrey and C. A. Whitman, Crystal Rectifiers, p. 3.

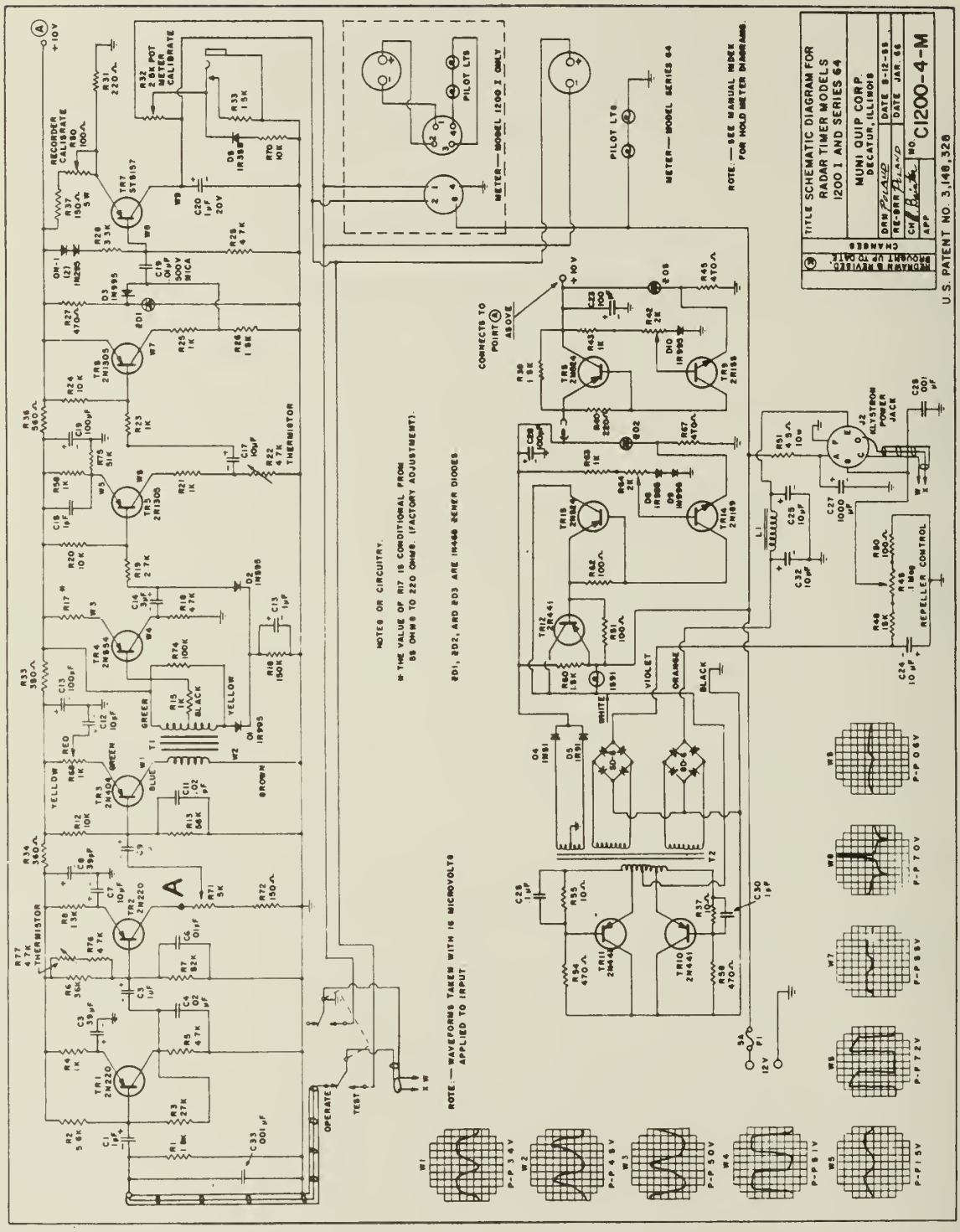
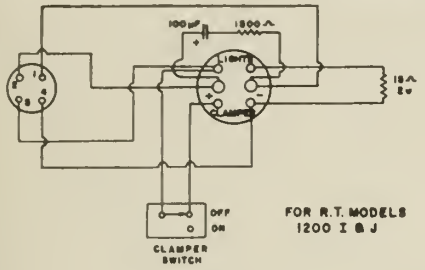
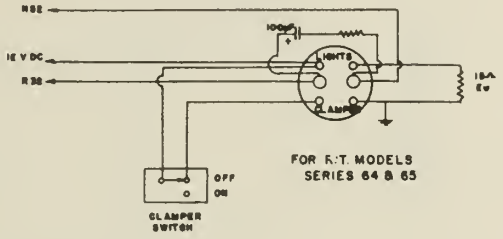

TITLE SCHEMATIC DIAGRAM FOR RADAR TIMER MODELS 1200 I AND SERIES 64
MUNI QUIP CORP. DECATUR, ILLINOIS
DRN DATE 8-12-65
RE-DRN DATE JAN. 66
CHK APP
NO C1200-4-M
U.S. PATENT NO. 3,140,328
NOTES ON CIRCUITRY:
* THE VALUE OF R17 IS CONDITIONAL FROM 85 OHMS TO 220 OHMS (FACTORY ADJUSTMENT).
ZD1, ZD2, AND ZD3 ARE IN468 ZENER DIODES
NOTE: — WAVEFORMS TAKEN WITH 16 MICROVOLTS APPLIED TO INPUT
NOTE: — SEE MANUAL INDEX FOR HOLD METER DIAGRAMS
METER — MODEL 1200 I ONLY
METER — MODEL SERIES 64
PILOT LTS
RECORDER CALIBRATE
10K POT METER CALIBRATE
THERMISTOR
REPELLER CONTROL
KLYSTRON POWER JACK
CONNECTS TO POINT A ABOVE
OPERATE
TEST
VIOLET
ORANGE
BLACK
WHITE
YELLOW
GREEN
BLUE
BROWN
RED

Figure 6

HOLD METER DIAGRAMS

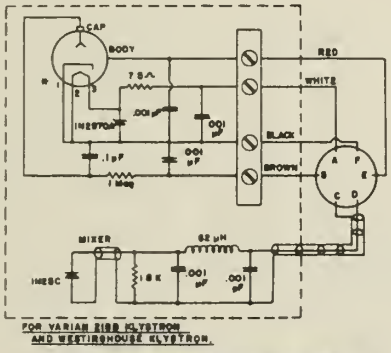


FOR R.T. MODELS
1200 I & J



FOR R.T. MODELS
SERIES 64 & 65

ANTENNA DIAGRAM



FOR VARIAN 8188 KLYSTRON
AND WESTINGHOUSE KLYSTRON.

*NOTE. — PINS 1 AND 2 ARE TIED INTERNALLY ON 8188.
ON WESTINGHOUSE THERE MUST BE A JUMPER WIRE
ON THE SOCKET FROM PIN 1 TO PIN 2.

CHANGES	TITLE HOLD METER AND ANTENNA DIAGRAMS FOR RADAR TIMERS.	
	MUNI QUIP CORP. DECATUR, ILLINOIS	
	DRN Piland	DATE 9-16-65
	RE-DRN Piland	DATE 1-2-66
	CH	NO. C1200-5-D
	APP	

Figure 7

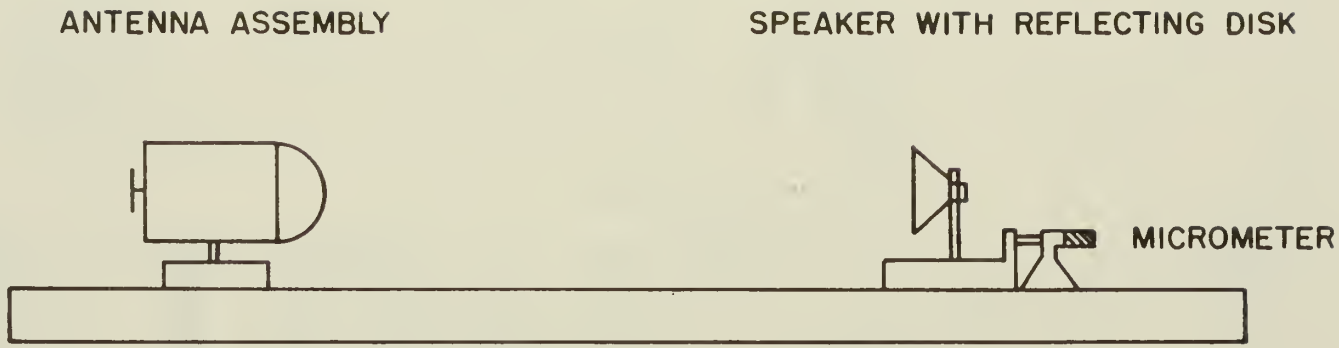


Figure 8. Schematic Diagram of Optical Bench

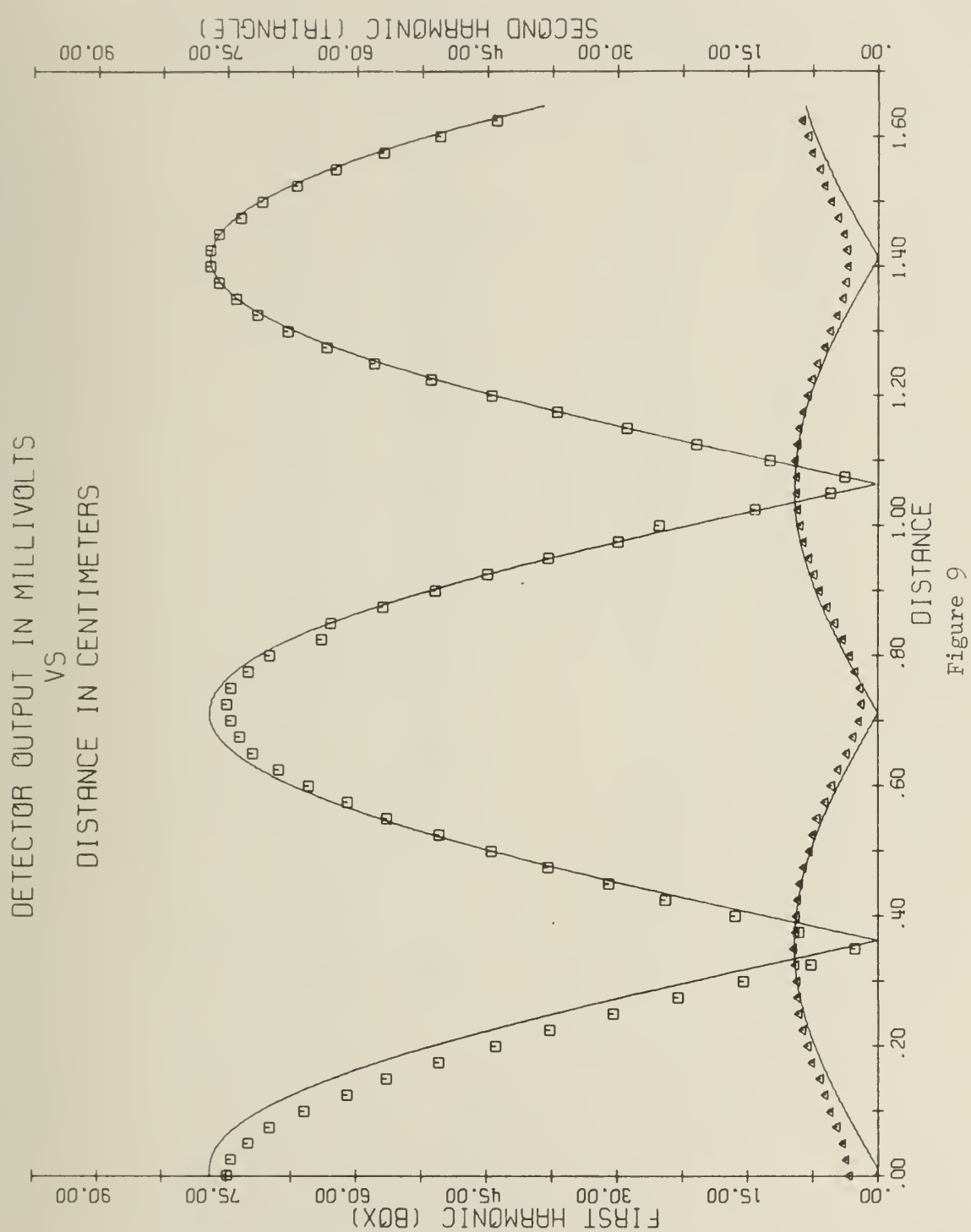

DETECTOR OUTPUT IN MILLIVOLTS
VS
DISTANCE IN CENTIMETERS
FIRST HARMONIC (BOX)
SECOND HARMONIC (TRIANGLE)
.00
15.00
30.00
45.00
60.00
75.00
90.00
.20
.40
.60
.80
1.00
1.20
1.40
1.60
DISTANCE

Figure 9

TABLE 1

First and Second Harmonic Voltages

Δd_0(cm)	V_1(mV)	V_2(mV)	Δd_0(cm)	V_1(mV)	V_2(mV)
0.000	75.0	3.7	0.825	67.0	4.5
0.025	75.0	3.9	0.850	63.0	5.5
0.050	73.0	4.4	0.875	57.0	6.5
0.075	70.0	5.2	0.900	51.0	7.4
0.100	66.0	5.9	0.925	45.0	8.2
0.125	61.0	6.7	0.950	38.0	8.7
0.150	57.0	7.1	0.975	30.0	9.3
0.175	51.0	8.2	1.000	27.5	9.7
0.200	44.0	8.8	1.025	14.5	10.0
0.225	38.0	9.3	1.050	5.6	10.0
0.250	31.0	9.9	1.075	3.8	10.0
0.275	23.0	10.0	1.100	12.5	10.5
0.300	15.5	10.2	1.125	20.5	10.0
0.325	7.9	10.3	1.150	29.0	9.7
0.350	2.8	10.5	1.175	37.0	9.1
0.375	9.5	10.2	1.200	45.0	8.5
0.400	16.5	10.2	1.225	52.0	8.0
0.425	24.5	10.0	1.250	58.0	7.2
0.450	31.0	9.7	1.275	64.0	6.5
0.475	38.0	9.1	1.300	68.0	5.8
0.500	45.0	8.4	1.325	72.0	5.1
0.525	51.0	8.0	1.350	74.0	4.3
0.550	57.0	7.5	1.375	76.0	4.2
0.575	61.0	6.5	1.400	77.0	3.8
0.600	66.0	5.7	1.425	77.0	4.3
0.625	69.0	4.8	1.450	76.0	4.4
0.650	72.0	3.8	1.475	74.0	5.4
0.675	74.0	2.9	1.500	71.0	5.9
0.700	75.0	2.4	1.525	67.0	6.8
0.725	75.0	2.1	1.550	63.0	7.3
0.750	75.0	2.3	1.575	57.0	8.4
0.775	73.0	2.9	1.600	50.0	8.8
0.800	70.0	3.6	1.625	44.0	9.5

5.2 Measurement of Allowable Distortion

If the amplifier output is sufficiently distorted, it is possible that the square-wave convertor may produce a waveform which is not at the fundamental frequency of the input signal. This might possibly lead to miscalibration of the radar unit. Let it be noted that not all forms of distortion can cause miscalibration. Only distortion which causes a change in the sign of the slope of the signal may cause miscalibration; for example, the waveform in Figure 10A. However, distortion caused by saturation or cut-off of an amplifier (clipping) cannot cause miscalibration; Figure 10B. The particular type of distortion of concern here is that generated by a vibrating tuning fork. Since the first and second harmonics are the only ones of significance for vibrating tuning forks (Section IV), measurements involving only second harmonic distortion were taken.

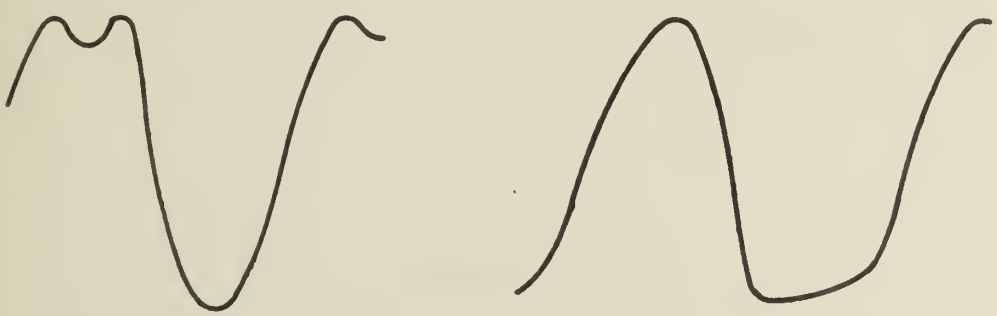


Figure 10

A Hewlett-Packard Model 203A Variable Phase Function Generator and a General Radio Model 1319A Oscillator were used to provide a signal in which the magnitudes of the first and second harmonics could be varied

independently. This signal was connected through a 5000 to 1 divider to W-X on Figure 6. W-X is the normal mixer input point. For these measurements, the mixer diode was removed and the klystron power supply disabled. Measurements were again taken at point A, Figure 6, using two frequency selective VTVM's.

With the RANGE control in the extreme clockwise position (maximum sensitivity), 13.5 mV RMS at point A was required to activate the meter. The frequency of the fundamental was approximately 1500 hertz and the output meter of the radar unit was indicating approximately 50 miles per hour as was expected. The second harmonic component of the signal was increased until the output meter began to move upscale. This was called the break point and the magnitude of the second harmonic at this point was recorded. If the amount of second harmonic is increased beyond the break point value, the indicated speed increased until it reached twice the original reading or approximately 100 miles per hour. The second harmonic voltage was also recorded at this point, which was called the leveling point. Table 2 shows the values obtained for various first harmonic voltages.

TABLE 2

V_1	V_2 at break point	V_2 at leveling point
13.5 mV RMS	10.0 mV RMS	16.0 mV RMS
30.0	19.5	31.0
100.0	62.0	110.0
300.0*	240.0	300.0

* Distortion caused by clipping becomes evident at 210 mV RMS.

It is obvious from Table 2 that V_2 must be greater than 1/2 V_1 in order to cause an improper speed indication. It should be mentioned that as V_1 increases beyond 210 mV RMS, the amount of second harmonic distortion necessary to cause an improper speed indication becomes close to 100 percent; that is $V_1 \simeq V_2$.

5.3 Approximation of δ for Tuning Forks

Three aluminum-alloy tuning forks are supplied by Muni Quip for the purpose of calibration. These forks vibrate at frequencies corresponding to 30, 45, and 65 miles per hour and these values are stamped on the respective forks. Normally, these forks are struck lightly and then placed directly in the path of the radiation. However, the second harmonic component generated by the mixing of these signals in the radar unit strongly depends upon the value of δ, the tine displacement. Therefore, to obtain the worst case conditions the forks were struck as hard as possible and the tine movement observed by means of a stroby lamp. It was found that none of the forks showed more than one millimeter total tine displacement, thus giving a maximum δ of 0.5 millimeters or 0.05 centimeters. It was also noted that the higher frequency forks had smaller displacements than the lower frequency forks. Table 3 shows the maximum first harmonic voltages obtainable at point A, Figure 6, due to reflections from the tuning forks.

TABLE 3

Fork	Serial No.	Maximum First Harmonic Voltage
30 mph	40649	1.5v
45 mph	40650	1.3v
65 mph	40651	0.8v

6. SUMMARY

When the operator wishes to calibrate the radar unit, he strikes the tuning fork, places it in front of the antenna, and looks for a steady reading on the output meter. When he obtains a steady reading, he adjusts the CALIBRATE control to bring the indicated speed into agreement with that stamped on the fork. Using the 30 mile per hour fork, it is possible to generate approximately 1.5 V RMS of first harmonic signal at point A, Figure 6. Since $\delta = 0.05$, Equation 16 indicates that $V_2(\max) = \frac{\pi\delta}{\lambda} V_1(\max)$ or 83.3 mV RMS of second harmonic signal can be produced if the fork is held in the proper position. If the RANGE control is set further clockwise than the midpoint, this is a sufficient second harmonic signal to cause the output meter to read double the true reading. However, to achieve this double reading, the operator must hold the tuning fork within 1/4 millimeter of the proper position.

Throughout the course of the experiment, the author has tried repeatedly to obtain a double reading with a hand-held tuning fork. Although one can observe the reduction of first harmonic component which indicates the fork is near the proper position, even an unsteady reading greater than that stamped on the tuning fork has never been achieved. This can be readily understood since it can be shown that the fork must be held in the plane perpendicular to the path of the radiation in addition to the fact it must be held exactly at the proper distance from the antenna.

Furthermore, as the operator seeks a more stable reading by moving the fork, he seeks a position where the voltage produced is

greater. This is equivalent to moving to a first harmonic maximum and thus away from the position which may cause a double reading.

7. CONCLUSIONS

On the basis of the measurements which have been made, it has been shown that the vibrating tuning fork approximates the signal caused by a moving target well enough to be used for calibration, and that it is practically impossible to miscalibrate a radar speed meter using the tuning fork method regardless of the position of the RANGE control. Furthermore, if someone with an extremely steady hand did manage to obtain a steady second harmonic reading when calibrating a radar unit, the miscalibration would be by a factor of two. This would cause a vehicle traveling at 60 miles per hour to register on the meter as 30 miles per hour. A miscalibration of this magnitude would be noticed immediately by the operator and result in recalibration.

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APPENDIX A

DOPPLER SHIFT[1,2]

Given incident electromagnetic radiation of frequency f_0 in a stationary medium, propagating to the left with velocity c, the speed of light. A reflecting surface S, is moving to the right with velocity v. All wavelengths, frequencies, and velocities are those observed by a stationary observer.

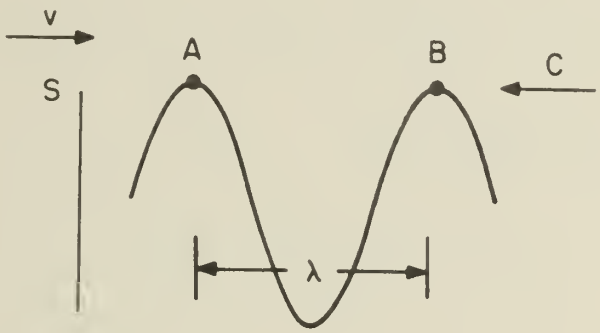

Figure 11

The wavelength of the radiation, λ, is given by c/f_0. Points A and B are successive maximum points in the electric field. Thus, points A and B are moving to the left at the velocity of light and are separated by λ, the wavelength. Point A strikes the reflecting surface at time t = 0, and at position x_0. At time t_1, point B strikes the surface. Since the reflecting surface and point B are approaching each other at velocity c + v, and the distance between them at t = 0 is λ,

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1. D. Fink, Radar Engineering, p. 287.

2. D. Luck, Frequency Modulated Radar, p. 7.

$$t_1 = \frac{\lambda}{c + v}$$

At time t_1, the reflecting surface S has moved to position $x_1 = x_0 + v(t_1)$ and point B of the wave is at S. Also at time t_1, point A has traveled from the point of reflection x_0 to $x_0 + ct_1$ since the reflected wave is now traveling to the right at the velocity of light. The distance between points A and B is the wavelength of the reflected wave λ'.

$$\lambda' = ct_1 - vt_1 \tag{19}$$

Substituting for t_1 from Equation 18.

$$\lambda' = \frac{(c - v)\lambda}{c + v} \tag{20}$$

Since $f_0 = \frac{\lambda}{c}$ and the frequency of the reflected wave f' equals λ'/c

$$f' = f_0 \frac{c + v}{c - v} \tag{21}$$

The difference frequency, $\Delta f = f' - f_0$

$$\Delta f = f_0 \frac{2v}{c - v} \tag{22}$$

If the incident electromagnetic radiation and the reflecting surface are moving in the same direction, the formula for the difference frequency is:

$$\Delta f = f_0 \frac{2v}{c + v}$$

The two expressions are for all practical purposes equal to $2f_0 v/c$ for all velocities yet attained of man-made vehicles. The following graph shows the Doppler shift (difference frequency) for various target speeds and electromagnetic wavelengths.

In the experimental work described in this paper, the incident frequency was 10,525 mHz. This corresponds to a difference frequency of 31.3674234 hertz per mile per hour. The error in disregarding the direction of travel is about .000015 per cent at 100 miles per hour.

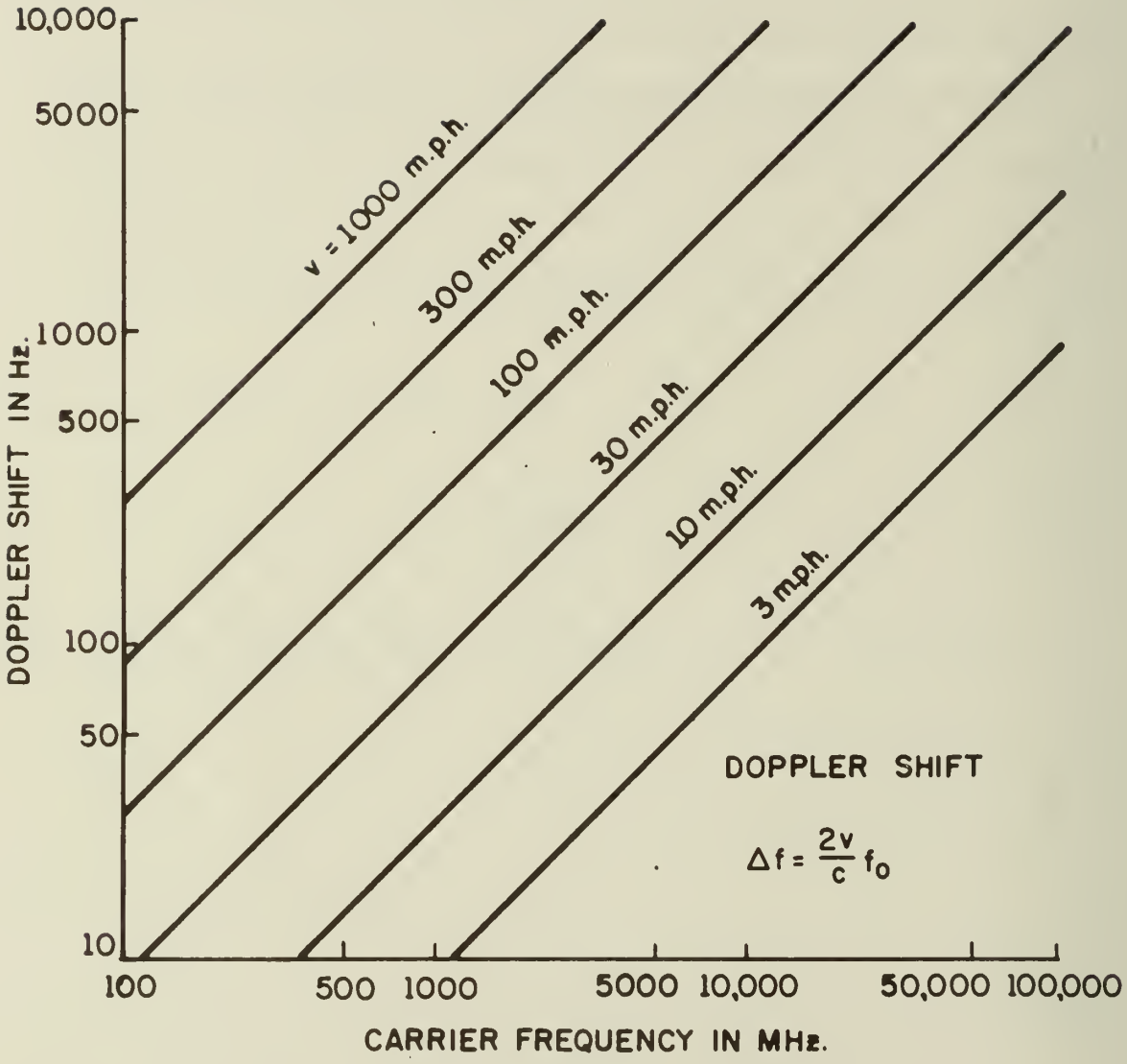
10,000
5000
1000
500
100
50
10
DOPPLER SHIFT IN Hz.
v = 1000 m.p.h.
300 m.p.h.
100 m.p.h.
30 m.p.h.
10 m.p.h.
3 m.p.h.
DOPPLER SHIFT
$\Delta f = \frac{2v}{c} f_o$
100
500
1000
5000
10,000
50,000
100,000
CARRIER FREQUENCY IN MHz.

Figure 12

APPENDIX B

ANGLE MODULATION AS APPLIED TO DOPPLER EFFECT

Angle modulation is defined to be modulation of the argument of the trigonometric function. That is, the signal $S = A \cos \phi$ is angle modulated if ϕ is modulated. Both frequency and phase modulations are special cases of angle modulation. In the case of an unmodulated signal, $\phi = \omega_c t + \theta_0$, where ω_c is the angular carrier frequency and θ_0 is some phase angle. The instantaneous angular frequency Ω is defined as $\partial\theta/\partial t$; for an unmodulated signal $\Omega = \omega_c$.

Now, frequency modulate the signal by causing the instantaneous angular frequency Ω to vary.

$$\text{Let} \quad \Omega = \omega_c + \Delta\omega f(t) \tag{24}$$

where ω_c is the carrier frequency, $\Delta\omega$ is the maximum frequency deviation, and $f(t)$ is a function of time with continuous first derivative. It follows that,

$$\phi = \int \Omega dt = \int [\omega_c + \Delta\omega f(t)]\, dt \tag{25}$$

$$\phi = \omega_c t + \Delta\omega \int f(t)\, dt \tag{26}$$

$$\therefore \quad S = A \cos [\omega_c t + \Delta\omega \int f(t)\, dt] \tag{27}$$

Take the unmodulated signal, $S = A \cos (\omega_c t + \theta_0)$ and cause θ_0 to vary as $\Delta\omega \int f(t)\, dt$. Then the resulting signal is the same as expressed by Equation 27.

Now, consider the Doppler shift as a frequency modulation of the carrier. In Equation 27, $\Delta\omega$ is the angular difference frequency, and $f(t) = 1$. Applying the results from above, a difference frequency of $\Delta\omega$ is equivalent to a continuous change of phase of $\Delta\omega$ radians per second.

$$S = \cos(\omega_c t + \Delta\omega t) \tag{28}$$

COMPUTER PROGRAM

THE PROGRAM REPRODUCED HERE IS THAT USED TO PRODUCE THE GRAPH OF THE FIRST AND SECOND HARMONICS SHOWN IN FIGURE 9. FOLLOWING THE PROGRAM IS THE PRINT OUT WHICH RESULTS. THE AVERAGED DATA SHOWN HERE IS THE SAME AS IS PLOTTED IN FIGURE 9.

MAIN PROGRAM

```
      CALL CPB1
      DIMENSION  DATA(10,2,66),AV(2,66),DD(66)
      RIT 7,1,N
    1 FORMAT(I2)
      RIT 7,2,(((DATA(I,J,K),J=1,2),K=1,66),I=1,N)
    2 FORMAT(2F10.2)
      CALL AVER(N,DATA,AV)
      CALL MATCH(AV,FMAX,SMAX,DD,THETA,THETA1,THETA2)
      Z = ABSF(THETA1) -ABSF(THETA2)
      IF(ABSF(Z)-.2)  67,100,100
   67 CALL GRAPH(THETA,DD,AV,FMAX,SMAX)
      GO TO 200
  100 CONTINUE
      WOT 6,5
    5 FORMAT(46H BAD MATCH IN THETA  *** PROGRAM TERMINATED ***)
  200 CONTINUE
      END
```

PROGRAM FOR SUBROUTINE AVER

THIS SUBPROGRAM AVERAGES UP TO 10 DATA SETS AND CORRECTS FOR ERRORS IN METER CALIBRATION.

```
      SUBROUTINE AVER(N,DATA,AV)
      DIMENSION DATA(10,2,66),AV(2,66)
      FN = N
      DO 1 L = 1,N
      DO 1 M = 1,66
    1 DATA(L,2,M) = 0.919*DATA(L,2,M)
      DO 10 I = 1,2
      DO 10  J = 1,66
      DO 15  K = 1,N
   15 AV(I,J) = AV(I,J) + DATA(K,I,J)
   10 AV(I,J) = AV(I,J)/FN
      RETURN
```

```
      END
PROGRAM FOR SUBROUTINE  MATCH
THIS SUBPROGRAM FINDS THE EXPERIMENTAL VALUE OF DELTA AND ALSO
EQUATION 17.

FINDS THE VALUE OF THETA TO BE SUBSTITUTED INTO THE THEORETICAL

      SUBROUTINE MATCH(DATA,FMAX,SMAX,DD,THETA,THETA1,THETA2)
      DIMENSION  DATA(2,66),DD(66)
      PI = 3.14159
      FMAX = 0.0
      SMAX = 0.0
      FMIN = 100.
      SMIN = 100.
      WOT 6,7
    7 FORMAT(1H1,20X,13HAVERAGED DATA)
      DO 10 K=1,66
      FK =K-1
      DD(K) = 0.025*FK
      WOT 6,6,DD(K),DATA(1,K),DATA(2,K)
    6 FORMAT(3F20.5)
      IF(FMAX-DATA(1,K)) 12,13,13
   12 FMAX=DATA(1,K)
   13 IF(SMAX-DATA(2,K)) 14,15,15
   14 SMAX=DATA(2,K)
   15 IF(FMIN-DATA(1,K)) 17,16,16
   16 FMIND=DD(K)
      FMIN=DATA(1,K)
   17 IF(SMIN-DATA(2,K)) 10,18,18
   18 SMIND=DD(K)
      SMIN=DATA(2,K)
   10 CONTINUE
      WOT 6,100
  100 FORMAT(//////,10X,7HMAXIMUM,14X,7HMINIMUM,14X,8HDISTANCE)
      WOT 6,4,FMAX,FMIN,FMIND,SMAX,SMIN,SMIND
    4 FORMAT(     /3F20.5,//,3F20.5)
      EXDELT =(2.805*SMAX)/(FMAX*PI )
      WOT 6,2,EXDELT
    2 FORMAT(1H ,///,31HEXPERIMENTAL VALUE OF DELTA IS ,F10.5,//)
      B =4.0*PI/2.805
      THETA1 = -B*FMIND          +PI/2.
      THETA2 =        -B*SMIND
      DO 65 I=1,2
      IF(I-2)37,38,38
   37 THETA=THETA1
      GO TO 39
   38 THETA=THETA2
   39 DO 59 J =1,5
      IF(THETA) 40,50,50
   40 IF(PI/2. +THETA)41,60,60
   41 THETA=THETA +PI
      GO TO 59
   50 IF(THETA -PI/2.) 60,71,71
   71 THETA=THETA -PI
   59 CONTINUE
   60 IF(I-2) 61,62,62
```

```
   61 THETA1=THETA
      GO TO 65
   62 THETA2=THETA
   65 CONTINUE
      THETA=(THETA1+THETA2)/2.0
      WOT 6,3,THETA1,THETA2,THETA
    3 FORMAT(9H THETA1 =,F10.5,//,9H THETA2 =,F10.5,//,8H THETA =
     1 ,F10.5)
      RETURN
      END
```

PROGRAM FOR SUBROUTINE GRAPH

THIS SUBPROGRAM GENERATES THE THEORETICAL CURVES FROM EQUATION 17 AND PLOTS THEM ALONG WITH THE AVERAGED DATA.

```
      SUBROUTINE  GRAPH(THETA,DD,DATA,FMAX,SMAX)
      DIMENSION  DATA(2,66),FHARM(660),SHARM(660),DD(66),D(660),
     1 TX(2),TY1(2)
      PI = 3.14159
      B =4.0*PI/2.805
      DO 20 I=1,660
      FI=I-1
      D(I)=.0025*FI
      ANG = THETA +B*D(I)
      FHARM(I)=ABSF(FMAX*COS(ANG))
   20 SHARM(I)= ABSF(SMAX*SIN(ANG))
      CALL CCP4SC(D,10.0,660,1,TX)
      CALL CCP4SC(FHARM,8.0,660,1,TY1)
      CALL CCP1PL(2.0,2.0,-3)
      CALL CCP5AX(0.0,0.0,20HFIRST HARMONIC (BOX),20,6.5,90.,TY1)
      CALL CCP5AX(0.0,0.0,8HDISTANCE,-8,08.5,0.0,TX)
      CALL CCP5AX(8.5,0.0,26HSECOND HARMONIC (TRIANGLE),-26,6.5,
     1 90.0,TY1)
      CALL CCP6LN(D,FHARM,660,1,TX,TY1)
      CALL CCP6LN(D,SHARM,660,1,TX,TY1)
      DO 30 I =1,66
      Y1=(DATA(1,I)-TY1(1))/TY1(2)
      Y2=(DATA(2,I)-TY1(1))/TY1(2)
      X = (DD(I)-TX(1))/TX(2)
      CALL CCP2SY(X,Y1,0.075,0,0.0,-1)
      CALL CCP2SY(X,Y2,0.075,2,0.0,-1)
   30 CONTINUE
      CALL CCP2SY(2.0,6.5,0.15,29HDETECTOR OUTPUT IN MILLIVOLTS,
     1 0.0,29)
      CALL CCP2SY(2.0,6.25,0.15,16H                VS,0.0,16)
      CALL CCP2SY(2.0,6.0,0.15,26H   DISTANCE IN CENTIMETERS,0.0,
     1  26)
      CALL CCP1PL(12.0,0.0,-3)
      RETURN
      END
```

AVERAGED DATA

DISTANCE	FIRST HARMONIC	SECOND HARMONIC
.00000	75.00000	3.30840
.02500	74.50000	3.63005
.05000	72.50000	3.99765
.07500	70.00000	4.73285
.10000	66.00000	5.42210
.12500	61.00000	6.06540
.15000	56.50000	6.61680
.17500	50.50000	7.53580
.20000	44.00000	8.04125
.22500	37.75000	8.59265
.25000	30.50000	9.05215
.27500	23.00000	9.19000
.30000	15.50000	9.28190
.32500	7.70000	9.46570
.35000	2.65000	9.64950
.37500	9.10000	9.41975
.40000	16.50000	9.37380
.42500	24.50000	9.19000
.45000	31.00000	8.96025
.47500	38.00000	8.54670
.50000	44.50000	7.85745
.52500	50.50000	7.44390
.55000	56.50000	6.98440
.57500	61.00000	6.06540
.60000	65.50000	5.33020
.62500	69.00000	4.54905
.65000	72.00000	3.58410
.67500	73.50000	2.84890
.70000	74.50000	2.13667
.72500	75.00000	1.83800
.75000	74.50000	2.02180
.77500	72.50000	2.59617
.80000	70.00000	3.17055
.82500	64.00000	3.99765
.85000	63.00000	4.96260
.87500	57.00000	5.83565
.90000	51.00000	6.70870
.92500	45.00000	7.35200
.95000	38.00000	7.94935
.97500	30.00000	8.50075
1.00000	25.25000	8.91430
1.02500	14.25000	9.19000
1.05000	5.55000	9.32785
1.07500	3.90000	9.32785
1.10000	02.50000	9.51165

1.12500	21.00000	9.23595
1.15000	29.00000	9.05215
1.17500	37.00000	8.59265
1.20000	44.50000	8.04125
1.22500	51.50000	7.58175
1.25000	58.00000	6.93845
1.27500	63.50000	6.11135
1.30000	68.00000	5.46805
1.32500	71.50000	4.68690
1.35000	74.00000	4.04360
1.37500	76.00000	3.67600
1.40000	77.00000	3.40030
1.42500	77.00000	3.53815
1.45000	76.00000	3.85980
1.47500	73.50000	4.54905
1.50000	71.00000	5.37615
1.52500	67.00000	6.11135
1.55000	62.50000	6.66275
1.57500	57.00000	7.53580
1.62500	50.50000	8.04125
1.62500	44.00000	8.73050

MAXIMUM FIRST HARMONIC VOLTAGE IS 77.00000 MILLIVOLTS

MINIMUM FIRST HARMONIC VOLTAGE IS 2.65000 MILLIVOLTS

POSITION OF FIRST HARMONIC MINIMUM IS 0.35000 CENTIMETERS

MAXIMUM SECOND HARMONIC VOTAGE IS 9.64950 MILLIVOLTS
MINIMUM SECOND HARMONIC VOTAGE IS 1.83800 MILLIVOLTS

POSITION OF SECOND HARMONIC MINIMUM IS 0.72500 CENTIMETERS

EXPERIMENTAL VALUE OF DELTA IS 0.11189 CENTIMETERS

THETA1 = 0.00280
THETA2 = -0.10640
THETA = -0.05180

NOV 28 1972

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